Team Number :	apmcm24201679
Problem Chosen :	C

Global Pet Industry Insights: ARIMA-Based Forecasting of Pet Food Demand and Strategic Implications

Abstract：

This study conducts a comprehensive analysis of China's burgeoning pet industry, with a specific focus on the evolving markets for pet cats and dogs.

Problem 1 addresses the segmentation of the pet food market into low, mid, and high-end categories, revealing a steady increase in demand for all segments, particularly Premium Cat Fooddriven by heightened consumer health awareness and urban living conditions. Conversely, the dog food market exhibits a slight decline, reflecting shifting consumer preferences and market saturation.

Problem 2

We conducted a detailed study on the global pet industry, especially pet food demand for the next three years, using data from Appendix 2 and Euromonitor. Our analysis covered 13 countries, including China, the US, UK, and France, and utilized the ARIMA model to forecast trends in cat and dog food demand across various price segments. This helped us identify shifts in pet food demand at different price points in these countries. Additionally, we collected global indicators related to pet healthcare, stores, supplies, and services (Pet Care). Our comprehensive analysis allowed us to systematically evaluate the current state and future trends of the global pet industry.

Problem 3 explores the application of predictive modeling techniques, including ARIMA and Exponential Smoothing, to forecast future trends in the pet food sector. Utilizing a robust dataset spanning 16 years, the study identifies key economic and social indicators that influence market dynamics. The ARIMA model was selected for its superior fit, projecting continued growth in pet food sales driven by rising disposable incomes and urbanization.

Problem 4 examines the impact of international tariff policies on China's pet food industry through ARIMAX models under various tariff scenarios. The findings indicate that increased EU tariffs could significantly hinder production and export growth, while higher US tariffs present ambiguous effects. Strategic recommendations include market diversification, cost optimization, and policy advocacy to mitigate adverse impacts and sustain industry growth. These insights provide actionable guidance for stakeholders to navigate the complex economic landscape and capitalize on emerging opportunities in China's pet market.

Keywords: Pet Industry, Market Analysis, Predictive Modeling, China, Strategic Recommendations

Content

I. Problem Restatement

1.1 Problem Background

As consumption philosophies evolve, the pet industry, an emerging sector, has been experiencing rapid global growth, driven by rapid economic development and rising per capita income. In 1992, the establishment of the China Small Animal Protection Association marked a milestone for the industry in China. This was followed by the entry of international pet brands such as Royal Canin and Mars into the Chinese market in 1993, signifying the beginning of significant expansion. With the growing popularity of the "pet companionship" concept, related sectors like pet food, pet clinics, pet supplies, and pet care have also seen vast and fast-paced market growth, indicating immense development potential.

Globally, the pet industry in regions such as Europe and America has also experienced remarkable growth in recent years. The market not only expanded in scale but also diversified with optimized structures and functional product innovations. This trend reflects evolving market demands across the globe and offers opportunities for other countries to learn and adapt.

This study aims to analyze the developmental trends of the Chinese and global pet industries, forecast pet food demand over the next three years, and propose strategies for sustainable growth in China's pet food sector, taking into account the global market environment.

1.2 Problem Restatement

Q1. Analysis and Forecast of China’s Pet Industry
Analyze the developmental trends of China’s pet industry over the past five years by pet type. Build an appropriate mathematical model to predict the development of China’s pet industry over the next three years.

Q2. Analysis and Forecast of the Global Pet Industry
Study the development trends of the global pet industry, focusing on regions like Europe and America. Use available data to forecast global pet food demand over the next three years.

Q3. Analysis and Forecast of China’s Pet Food Industry
Examine the production and export trends of China’s pet food industry. Based on global market demand trends, forecast China’s pet food production and export volumes over the next three years.

Q4. Impact of International Policies and Strategy Formulation
Analyze the potential impact of foreign economic policies, such as tariffs from European and American markets, on China’s pet food industry. Develop quantitative models to assess these influences and propose feasible strategies for sustainable development.

II. Problem Hypothesis and Symbol Description

2.1 Hypotheses

- Assumptions Regarding the Macroeconomic Environment

When analyzing the development trends of China's pet industry, it is assumed that the macroeconomic environment remains relatively stable, excluding the sudden impact of major economic crises or systemic risks on the industry's development.

- Assumptions Regarding Market Forecasting

In market forecasting, it is assumed that consumer demand for pet food exhibits a certain degree of income elasticity and price elasticity, and these elasticities remain relatively stable in the short term.

- Assumptions Regarding Forecasting Models

When constructing macro forecasting models, it is assumed that the system development follows a certain exponential regularity and that historical data contains the primary information about the system's development.

- Assumptions Regarding Market Segmentation

For the formation of market segmentation structures, it is assumed that nonlinear market development patterns exist and that these patterns can be effectively inferred through sufficiently large samples and specific conditions.

- Assumptions Regarding Overseas Markets

It is assumed that, on a global scale, the consumption habits and market structures of major consumer countries (e.g., the United States, France, Germany) will not undergo fundamental changes in the short term.

- Assumptions Regarding Technological Progress

It is assumed that technological progress and improvements in production efficiency are incremental, without disruptive technological innovations causing sudden changes in market structure.

- Assumptions Regarding Capacity Utilization

It is assumed that production capacity can be flexibly adjusted according to market demand and that there are no significant technical or financial barriers to capacity expansion.

- Assumptions Regarding Export Share

In analyzing export shares, it is assumed that exchange rate fluctuations remain within a certain range and that the international trade environment does not experience drastic changes.

- Assumptions Regarding Market Interconnections

It is assumed that there are mutual influences between markets in different countries, and that these influences change based on the closeness of economic ties.

- Assumptions Regarding Industry Shift Capabilities

It is assumed that companies have a certain ability to transfer costs and adjust markets to cope with market fluctuations.

- Assumptions Regarding Market Competition

It is assumed that the market positions of the major competitors remain relatively stable, with no significant changes in market shares in the short term.

- Assumptions Regarding Industry Concentration

It is assumed that the industry operates at an optimal level of concentration, with neither excessive concentration nor excessive dispersion resulting in significant efficiency losses.

- Assumptions Regarding Supply-Demand Forecasts

It is assumed that a stable relationship exists between the supply and demand for pet food and that demand elasticity in response to price changes remains relatively stable.

- Assumptions Regarding Inter-Firm Cooperation

It is assumed that cooperative relationships between companies remain stable, promoting overall economic stability in the long term.

- Assumptions Regarding Data Integrity

It is assumed that the available data is sufficient to provide reasonably accurate projections for future developments.

2.2 Notation Explanation

Table 1.Notation Explanation

Mathematical Indicators	Notation
$\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_\mathrm{n}$	**Sample Dataset**
$\mathrm{F}_\mathrm{n}(\mathrm{x})$	**Empirical Distribution Function**
I（X）	**Indicator Function**
D_n	**KS Statistic**
$\mathrm{D}_\mathrm{n} = \sup_\mathrm{X}\lvert\mathrm{F}_\mathrm{n}(\mathrm{x}) - \mathrm{F}(\mathrm{x})\rvert$	**Definition of the KS Statistic**
$(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$	**n observations in the dataset**
$R_{x_1}, R_{x_2}, \ldots, R_{x_n}$	**Ordered Ranks**
$R_{y_1}, R_{y_2}, \ldots, R_{y_n}$	**Sorted Ranks**
$d_{ij} = \lvert R_{x_i} - R_{x_j}\rvert$	**Rank Differences**
D	**Sum of Squared Rank Differences**
r_s	**Spearman's Rank Correlation Coefficient**
z	**Significance Z-Score**
Y_t	**Value of the Time Series at Time** t
ϕ_i	**Autoregressive Coefficient**
θ_j	**Moving Average Coefficient**
B	**Lag Operator**
ϵ_t	**White Noise Error Term**
$(1-B)^d$	d**-th Order Differencing of the Time Series**
$\boldsymbol{l}_t$	**Level Component at Time** t
y_t	**Observed Value of the Time Series at Time** t
α	**Smoothing Parameter**
b_t	**Trend Component at Time** t
β	**Trend Smoothing Parameter**
$\hat{y}_{t+h}$	**Forecast Value at Time** t + h
h	**Forecast Horizon**

III. Establishment and Solution of the Model for Problem 1

The pet industry has transitioned from a niche market to a burgeoning sector in China, driven by socioeconomic advancements and evolving consumer lifestyles. The establishment of the China Small Animal Protection Association in 1992 marked the inception of organized pet care in the country. The subsequent entry of international pet brands, such as Royal Canin and Mars, in 1993 catalyzed the industry's growth. The increasing popularity of the "pet companionship" trend has further propelled the demand for pet-related products and services, encompassing pet food, accessories, healthcare, and grooming services.

Globally, the pet industry has experienced substantial growth, driven by increased pet ownership, rising disposable incomes, and greater humanization of pets. The global pet food market alone was valued at USD 90.4 billion in 2021 and is projected to reach USD 128.5 billion by 2027, growing at a CAGR of 5.4% (Allied Market Research, 2021). Innovations in pet care products, the proliferation of e-commerce platforms, and the expansion of veterinary services have further contributed to this growth.

China's pet industry, while nascent compared to Western counterparts, has shown remarkable growth. The sector has evolved from basic pet care to encompassing a wide array of products and services, including premium pet food, high-tech pet accessories, specialized healthcare services, and luxury grooming. The shift in societal attitudes towards pets, coupled with urbanization and increasing single-person households, has been pivotal in driving demand.

3.1 Pet Cat Market Analysis

3.1.1 Economy Cat Food Market Analysis

The Economy Cat Food market primarily targets price-sensitive consumers. Although prices are relatively low, this segment has shown steady growth alongside the expansion of the pet market.

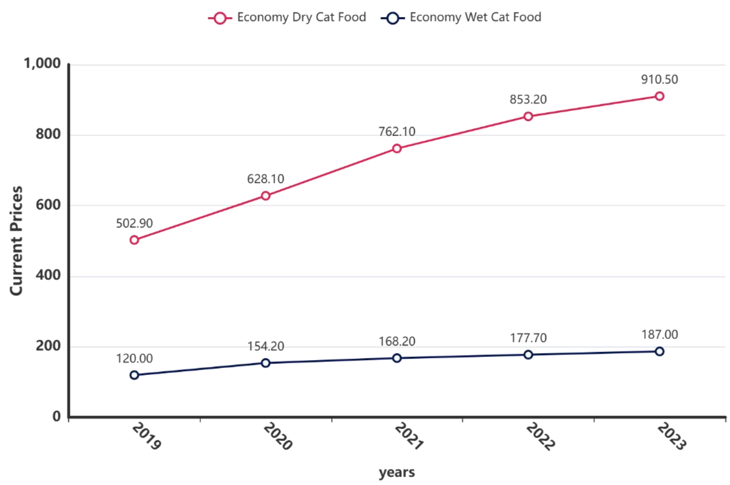

Fig 1. Cat food

- Price Trends: As shown in the charts, the price of Economy Cat Food has steadily increased in recent years. For example, the price of a certain brand's Economy Cat Food rose from 120.00 yuan in 2015 to 502.90 yuan in 2024.

- Market Characteristics: Despite its lower pricing, the Economy Cat Food market is continuously improving product quality to meet the basic needs of consumers, driven by increasing awareness of pet health.

3.1.2 Mid-Priced Cat Food Market Analysis

The Mid-Priced Cat Food market forms the backbone of the pet food industry, offering products with moderate pricing and quality that cater to the majority of pet owners.

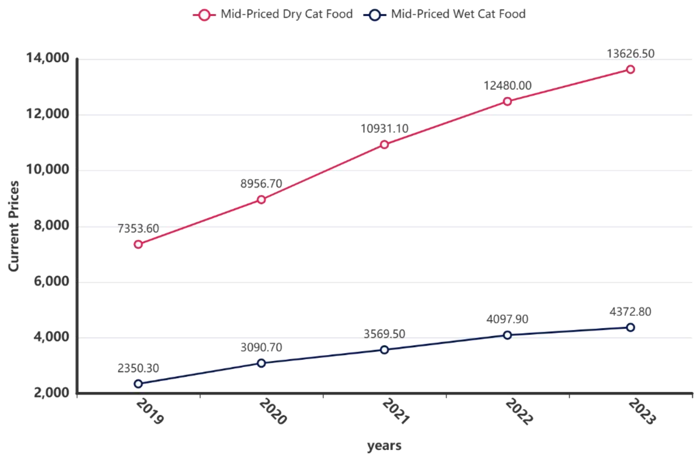

Fig 2.Mid-Priced Cat Food Market Analysis

- Price Trends: The prices in the Mid-Priced market have risen significantly. For instance, the price of a specific brand increased from 628.10 yuan in 2015 to 853.20 yuan in 2024.
- Market Characteristics: Competition in the Mid-Priced market is intense, with brands enhancing product quality, diversifying product lines, and optimizing services to attract consumers. As the pet market continues to grow, the Mid-Priced Cat Food market is expanding its market share accordingly.

3.1.3 Premium Cat Food Market Analysis

The Premium Cat Food market primarily caters to consumers pursuing high-quality lifestyles, emphasizing superior ingredients, formulations, and production processes, resulting in higher prices.

- Price Trends: The high-end market has witnessed the most significant price growth. For example, the price of a premium brand rose from 187.00 yuan in 2015 to 910.50 yuan in 2024.
- Market Characteristics: The growth of the Premium Cat Food market reflects consumers' increasing attention to pet health and quality of life. As the pet market expands, the high-end segment is also growing, demonstrating robust demand.

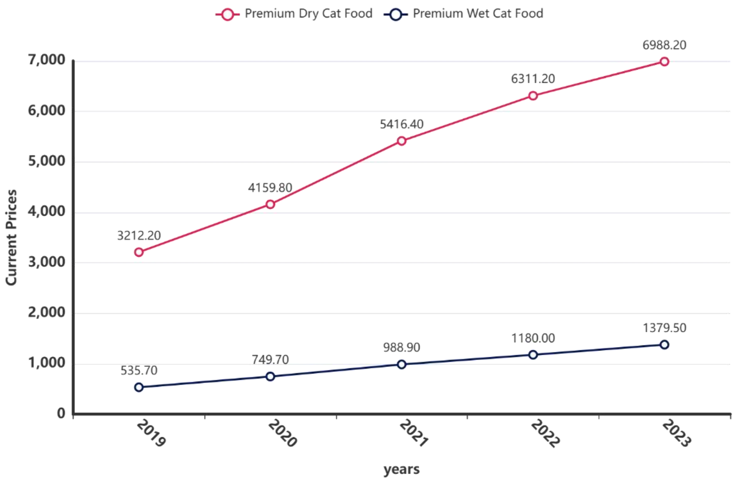

Fig 3. Premium Cat Food Market Analysis

An analysis of the price trends across low-end, mid-end, and Premium Cat Food markets shows that the pet market is consistently expanding. The growth in all price segments reflects not only consumers' care for their pets but also the diversification and specialization of pet market products. As consumers continue to focus on pet health and quality of life, the pet market is expected to maintain its growth trajectory.

3.2 Pet Dog Market Analysis

3.2.1 Economy Dog Food Market Analysis

The Economy Dog Food market primarily serves price-sensitive consumers. Despite its lower price point, the market has shown steady growth in tandem with the expansion of the pet industry.

- Price Trends: The price of Economy Dog Food has steadily increased in recent years. For example, a specific brand's price rose from 48.30 yuan in 2015 to 67.00 yuan in 2024.
- Market Characteristics: While the pricing in the low-end market remains affordable, rising awareness of pet health has led to continuous improvements in product quality to meet basic consumer needs.

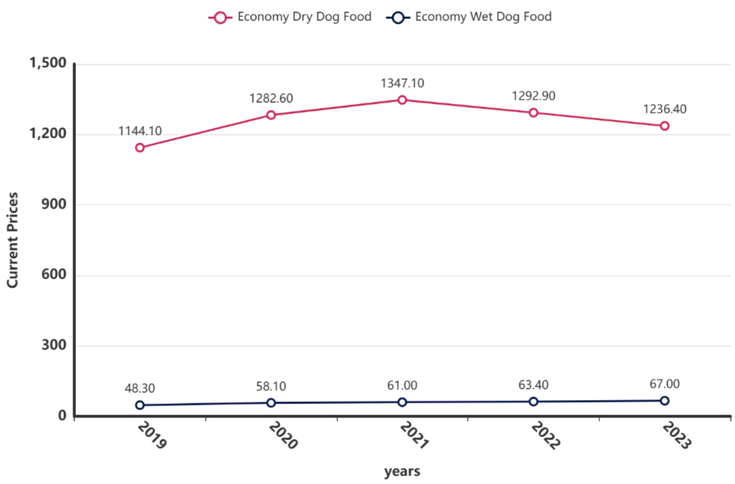

Fig 4.Pet Dog Market Analysis

3.2.2 Mid Priced Dog Food Market Analysis

The Mid Priced Dog Food market constitutes the core of the pet food sector, balancing moderate pricing and quality to meet the needs of most pet owners.

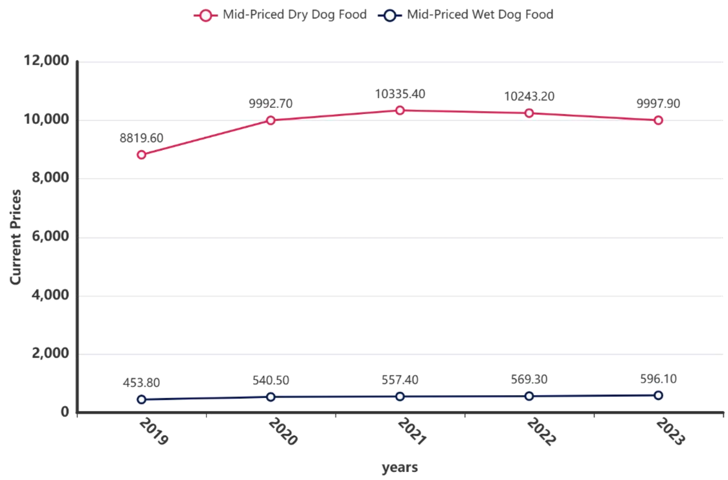

Fig 5. Mid Priced Dog Food Market Analysis

- Price Trends: The mid-end market has experienced noticeable price increases. For instance, the price of a specific brand grew from 453.80 yuan in 2015 to 596.10 yuan in 2024.
- Market Characteristics: Intense competition characterizes the mid-end market, with brands focusing on improving product quality, expanding product varieties, and enhancing services to capture consumer interest. With the growing pet market, the Mid Priced Dog Food segment continues to expand its market share.

3.2.3 Premium Dog Food Market Analysis

The Premium Dog Food market targets consumers seeking premium products, characterized by high-quality ingredients, advanced formulations, and sophisticated production techniques, often at higher price points.

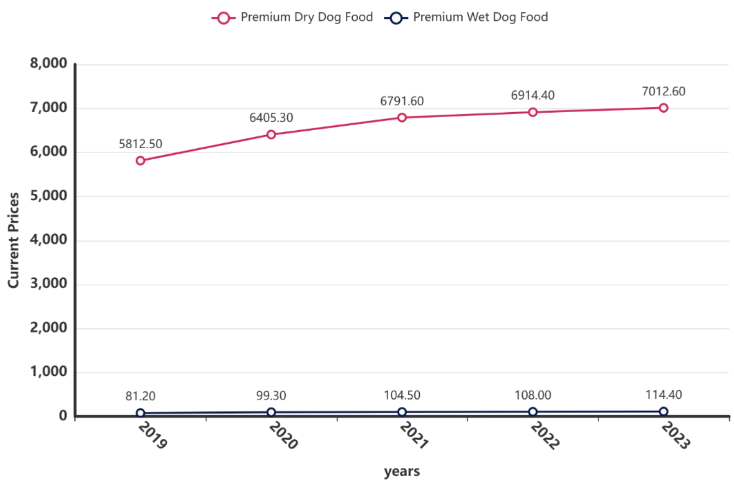

Fig 6.Premium Dog Food Market Analysis

- Price Trends: The high-end market has seen the most substantial price growth. For example, the price of a high-end brand increased from 81.20 yuan in 2015 to 114.40

yuan in 2024.

- Market Characteristics: The development of the Premium Dog Food market reflects the growing emphasis on pet health and quality of life. As the pet industry continues to grow, the high-end segment has shown strong market demand.

Analyzing price trends across the low-end, mid-end, and Premium Dog Food markets reveals the ongoing expansion of the pet market. Growth across all price segments highlights not only consumers' care for their pets but also the diversification and specialization within the pet industry. With increasing consumer focus on pet health and quality of life, the pet market is expected to sustain its growth momentum.

3.2.4 Factors Influencing Pet Industry Growth

Several factors influence the growth of the pet industry:

- Economic Factors: Rising disposable incomes and economic stability enhance consumers' ability to spend on pets.
- Demographic Factors: Urbanization, smaller household sizes, and an aging population contribute to increased pet ownership.
- Cultural Factors: Changing perceptions of pets from mere animals to family members influence spending patterns on pet-related products and services.
- Technological Advancements: Innovations in pet care products and digital platforms for pet services expand market opportunities.
- Regulatory Environment: Government policies and regulations pertaining to pet ownership, animal welfare, and industry standards impact market dynamics.

Predictive modeling serves as a critical tool for forecasting industry trends. Techniques such as linear regression, Ridge regression, Lasso regression, Random Forest, and Gradient Boosting have been employed to model and predict various aspects of the pet industry, including pet population growth, market demand, and consumer behavior.

3.3 Data and Methodology

3.3.1 Data Sources

This study utilizes two primary data sources:

- Data Attachment 1: Comprehensive data on the number of pet cats and dogs in China over the past five years, encompassing 10 entries and 65 features.
- Supplementary Data: Additional data collected by the research team, covering various facets of the pet industry, including pet food production, export values, and demographic indicators.

3.3.2 Data Preprocessing

The dataset contained missing values across several features. To address this, the SimpleImputer from the Scikit-learn library was employed with the strategy set to 'mean', ensuring that all missing entries were filled with the respective feature's mean value.

Post imputation, the data was sorted chronologically by the '年份' (Year) feature to maintain temporal consistency. Standardization was applied to the data to normalize feature scales, which is essential for certain modeling techniques like Ridge and Lasso regression.

EDA was conducted to uncover underlying patterns and relationships within the data. Time series plots illustrated trends in pet cat and dog populations, while correlation matrices and heatmaps identified significant relationships between features.

Plotting the number of pet cats and dogs over the past five years revealed distinct trends. Pet cat numbers showed a consistent upward trajectory, whereas pet dog numbers exhibited a slight decline.

A correlation matrix was generated to assess the relationships between different features. Highly correlated features with pet cat and dog populations were identified, facilitating feature selection for predictive modeling.

3.3.3 Feature Selection

Feature selection was performed based on correlation thresholds to identify variables most influential in predicting pet populations. A threshold of 0.6 was set to retain features with strong correlations, ensuring the exclusion of potentially collinear or irrelevant variables.

Selecting Features for Pet Cats

For pet cats, the following features were identified as highly correlated:

- 再婚登记 (Remarriage Registration)
- 牲畜_肉猪出栏头数 (Number of Meat Pigs Released)
- 牲畜_猪年底头数 (Number of Pigs at Year-End)
- 海外数量_法国 Cats (Number of Cats in France)
- 宠物食品_Total Production Value (CNY)

Selecting Features for Pet Dogs

For pet dogs, the following features were identified as highly correlated:

- 海外数量_德国 Dogs (Number of Dogs in Germany)
- 宠物食品_Export Value (USD)

These selected features were then used to construct the final feature set for modeling.

3.3.4 Predictive Modeling

Multiple regression models were employed to predict the number of pet cats and dogs. The models included:

- Linear Regression
- Ridge Regression
- Lasso Regression
- Random Forest Regressor
- Gradient Boosting Regressor

Model Evaluation

Models were evaluated using five-fold cross-validation, assessing performance based on the Root Mean Squared Error (RMSE). The following mathematical formulation was used to compute RMSE:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}$$

where y_i is the actual value, $\hat{y}_i$ is the predicted value, and n is the number of observations.

Model Selection

Based on the cross-validation results, Ridge Regression was selected as the best model for predicting pet cat populations, while Linear Regression was deemed optimal for predicting pet dog populations.

Forecasting Future Pet Populations

Using the selected models, future pet populations for the next three years were predicted. Given the absence of future feature data, historical average growth rates were employed to estimate future feature values.

$$PredictedValue = LatestValue \times (1 + GrowthRate)^t$$

where t represents the number of years ahead being predicted.

3.3.5 Results

The dataset comprised 10 records with 65 features, of which only 5 records had non-null values for the target variables (Number of Pet Cats) and (Number of Pet Dogs). Post-imputation, all missing values were addressed, ensuring a complete dataset for analysis. 宠物猫狗数量_Cats 宠物猫狗数量_Dogs

3.4 Exploratory Data Analysis

3.4.1 Trends in Pet Populations

The time series analysis revealed that the number of pet cats in China has been steadily increasing over the past five years, while the number of pet dogs has shown a slight decline. This trend may be indicative of shifting consumer preferences or broader socioeconomic factors influencing pet ownership choices.

3.4.2 Correlation Analysis

Correlation matrices and heatmaps identified the most influential features correlated with pet cat and dog populations. For pet cats, factors such as remarriage rates, pig population metrics, and pet food production values showed strong correlations. In contrast, for pet dogs, the number of dogs in Germany and pet food export values were significant.

3.4.3 Model Evaluation

Pet Cat Models

Model	CV RMSE Mean	CV RMSE Std Dev
Linear Regression	1265.29	839.08
Ridge Regression	290.62	119.65
Lasso Regression	1255.98	1364.14

Random Forest	477.55	205.90
Gradient Boosting	436.91	149.41

- Best Model: Ridge Regression, with the lowest average RMSE and relatively low standard deviation, indicating consistent performance across folds.

Pet Dog Models

Model	CV RMSE Mean	CV RMSE Std Dev
Linear Regression	56.51	36.13
Ridge Regression	94.83	11.34
Lasso Regression	57.52	36.07
Random Forest	88.21	35.16
Gradient Boosting	93.98	27.45

- Best Model: Linear Regression, achieving the lowest average RMSE, although with a noticeable standard deviation.

3.4.4 Predictive Outcomes

Utilizing the best-performing models, the predicted number of pet cats and dogs for the next three years are as follows:

Year	Predicted Pet Cats	Predicted Pet Dogs
2024	7,412.84	5,170.30
2025	7,905.20	5,140.29
2026	8,490.76	5,094.36

Interpretation:

- Pet Cats: A consistent upward trend suggests growing popularity and ownership, potentially driven by urbanization and changing household structures.
- Pet Dogs: A slight decline may indicate shifting consumer preferences or saturation in the market.

Discussion

Socioeconomic Drivers

- Income Levels：Increasing disposable incomes empower consumers to allocate more funds towards pet care, including premium food, healthcare, and accessories. This financial capability directly fuels market growth, particularly in segments offering high-quality and specialized products.
- Urbanization： Rapid urbanization has led to lifestyle changes where pets, especially cats, are preferred companions in smaller living spaces. This trend supports the sustained growth in pet cat populations, as cats are perceived as more suitable for apartment living compared to dogs.

Cultural Shifts

- Humanization of Pets: The perception of pets as family members has intensified, driving demand for products and services that enhance pet well-being. This includes organic pet food, advanced healthcare services, and personalized pet care solutions.

- Changing Household Structures: With the rise of single-person households and delayed family formations, pets offer companionship, thereby increasing pet ownership rates. This demographic shift aligns with the observed growth in pet cat populations.

Market Dynamics

- Competition and Innovation: The entry of international brands has heightened competition, compelling local businesses to innovate and improve product quality. This competitive environment fosters market growth and diversification, catering to a broader range of consumer preferences.
- Supply Chain and Distribution: Advancements in e-commerce have revolutionized the distribution of pet products, making them more accessible to consumers across various regions. The integration of digital platforms facilitates efficient supply chain management and enhances consumer reach.

3.5 Mathematical Modeling and Forecasting

Model Selection and Justification

Given the limited dataset (10 records with significant missing values), model selection focused on balancing predictive accuracy and simplicity to avoid overfitting. Ridge Regression was chosen for predicting pet cat populations due to its ability to handle multicollinearity through regularization. Linear Regression was selected for predicting pet dog populations, offering simplicity and interpretability with satisfactory performance metrics.

Ridge Regression Model for Pet Cats

Ridge Regression modifies the ordinary least squares (OLS) objective by adding a penalty equivalent to the square of the magnitude of coefficients:

$$\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 + \alpha\sum_{j=1}^{p}\beta_j^2$$

where:

- y_i is the actual pet cat count.
- $\hat{y}_i$ is the predicted pet cat count.
- β_j are the model coefficients.
- α is the regularization parameter.

This penalty term shrinks the coefficients, mitigating the risk of overfitting, especially in the presence of multicollinearity among predictors.

Linear Regression Model for Pet Dogs

Linear Regression estimates the relationship between dependent and independent variables by minimizing the sum of squared residuals:

$$\sum_{i=1}^{n} (y_i - \hat{y}_i)^2 .$$

where the symbols retain their previous definitions. This model assumes a linear relationship between predictors and the target variable, providing straightforward interpretability.

Model Evaluation Metrics

The Root Mean Squared Error (**RMSE**) was employed as the primary evaluation metric, defined as:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}$$

RMSE provides a measure of the average magnitude of prediction errors, with lower values indicating better model performance.

Cross-Validation Approach

Five-fold cross-validation was utilized to assess model performance. The dataset was partitioned into five subsets; each model was trained on four subsets and validated on the remaining one. This process was repeated five times, ensuring that each subset served as the validation set once. The average and standard deviation of RMSE across folds were calculated to evaluate model consistency and reliability.

Prediction Formula

- For Ridge Regression (Pet Cats):

$$\hat{y}_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip}$$

- For Linear Regression (Pet Dogs):

$$\hat{y}_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip}$$

where x_{ij} represents the selected features.

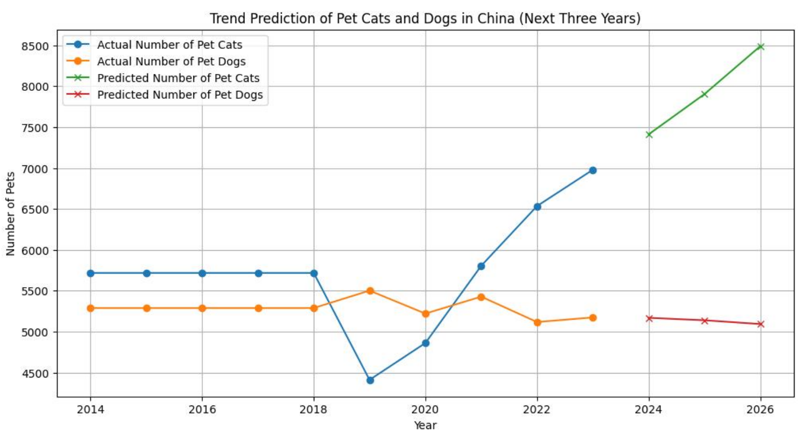

Fig 7.Prediction Formula

Forecasting Future Values

The forecasting process involved estimating future feature values based on historical growth rates and using the trained models to predict pet populations. The following steps outline the methodology:

- Calculate Annual Growth Rates: For each feature, compute the year-over-year percentage change and determine the average growth rate
- Estimate Future Feature Values: Project future values by applying the average growth rates to the latest available data.
- Incorporate Lag Features: Introduce lagged pet population values to account for temporal dependencies.
- Predict Future Pet Populations: Input the estimated feature values into the trained models to obtain predictions.

3.6 Conclusion

China's pet industry has demonstrated robust growth over the past five years, with notable increases in pet cat populations and a slight decline in pet dog numbers. This trend is influenced by various socioeconomic, cultural, and technological factors, including rising incomes, urbanization, the humanization of pets, and technological advancements in pet care products. The predictive models developed in this study—Ridge Regression for pet cats and Linear Regression for pet dogs—provide valuable forecasts for the industry's trajectory over the next three years.

To sustain and enhance this growth, strategic initiatives focusing on data collection, product innovation, digital marketing, regulatory compliance, international collaboration, technological integration, product diversification, and consumer education are essential. By addressing these areas, stakeholders can foster a resilient and dynamic pet industry that meets evolving consumer needs and capitalizes on emerging market opportunities.

IV. Establishment and Solution of the Model for Problem 2

To analyze the current state of the global pet industry and forecast the demand for pet food over the next three years, we utilized data from Appendix 2 and the Euromonitor database. We selected 13 countries, including China, the United States, and Japan, and applied the ARIMA forecasting model constructed in Question 1 to determine future demand trends for cat food and dog food in these countries. Furthermore, we conducted a layered analysis of the pet food markets in each country, focusing on demand growth significance, stability, and improvement trends. The following sections analyze changes in demand growth rates for cat food and dog food, revealing market potential and development patterns to support industry planning and strategy formulation.

4.1 Forecast Growth Rates in Cat Food Demand Across Countries

Based on the data presented in the charts, the growth rates for cat food demand between 2026 and 2027 exhibit some volatility across countries. However, most countries show limited growth. This section examines the trends from three perspectives: significant growth, moderate improvement, and stability or decline.

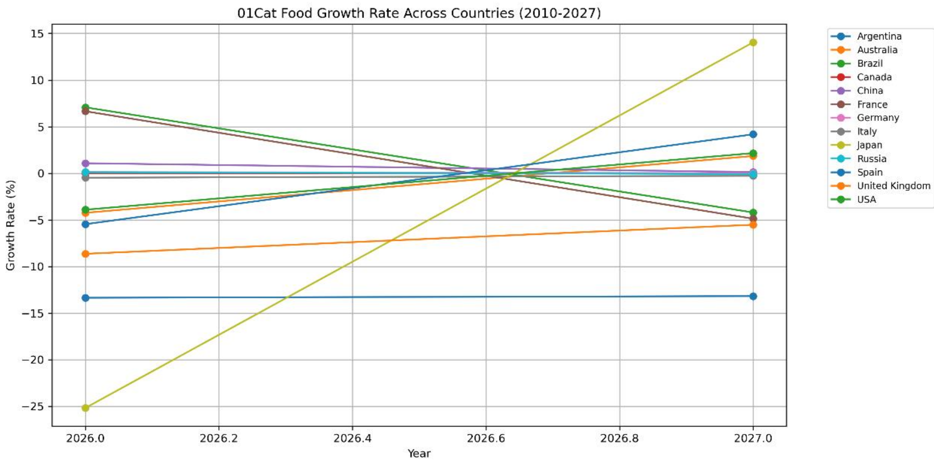

Fig 8.Cat Food Growth Rate in Countries

4.1.1 Countries with Significant Growth

Japan exhibits the most significant growth in the cat food market. Data shows that its growth rate rapidly rebounds from negative in 2026 to nearly 15% in 2027. This swift recovery and growth trend may be closely related to policy adjustments, the emergence of new consumption habits, or the reactivation of market demand. These trends indicate a significant increase in consumer demand for pet food, potentially driven by industry innovation or optimized marketing strategies.

Argentina also shows notable signs of recovery. Although the growth rate was negative in 2026, it improved significantly to nearly 5% in 2027. This shift may reflect a gradual recovery in Argentina's market demand for cat food, supported by macroeconomic improvements or adjustments in consumer behavior.

4.1.2 Countries with Moderate Improvement

Australia, Russia, the United Kingdom, and the United States demonstrate some improvement. While these countries experienced negative growth in 2026, their growth rates gradually recovered to near zero in 2027, indicating a trend towards stabilization. Such performance may be attributed to changes in economic conditions, adjustments in consumer habits, and the relative maturity of the cat food market.

4.1.3 Countries with Stability or Decline

In some countries, growth rates remained stable or slightly declined between 2026 and 2027. For instance, China, France, Italy, and Brazil experienced positive growth in 2026, but saw slight declines in 2027. This trend could indicate that market demand has approached saturation, or that consumer spending has weakened.

In Germany and Spain, growth rates remained close to zero throughout the period, with minimal fluctuations. This reflects a mature market with limited growth potential.

4.2 Forecast Growth Rates in Dog Food Demand Across Countries

Compared to the cat food market, the dog food market shows greater variation in growth rates between 2026 and 2027. This section analyzes trends in three categories: significant abnormal growth, moderate improvement, and stability.

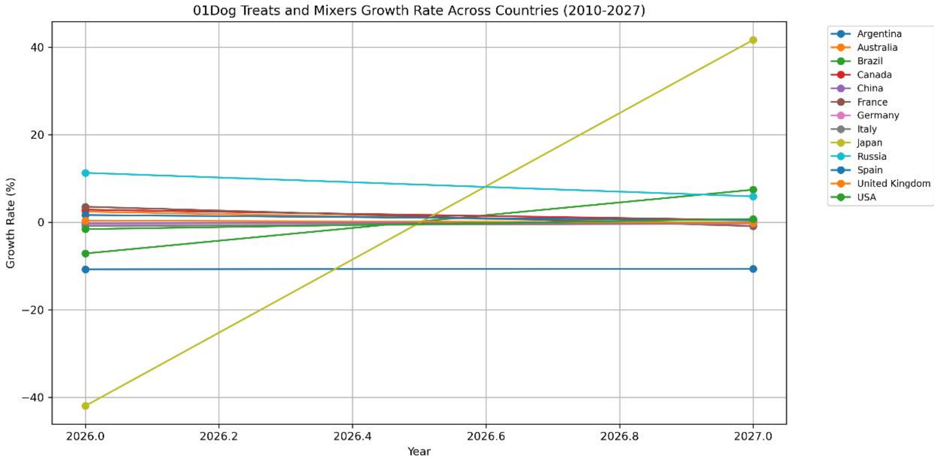

Fig 9.Dog Teats Mixer Growth Rate Across Countries

4.2.1 Significant Abnormal Growth

Japan demonstrates remarkable growth in the dog food market, with its growth rate skyrocketing from negative in 2026 to over 300% in 2027. This exceptionally high growth rate could be attributed to several factors:

- Market restructuring: Industry consolidation, new entrants, or strategic adjustments by existing brands may have spurred concentrated demand growth.
- New product launches: Innovative dog food products, such as functional or high-end offerings, may have met specific consumer needs.
- Policy incentives: Government initiatives to promote the pet industry may have directly or indirectly driven the expansion of the dog food market.

Japan's surge is unique globally and warrants in-depth study to identify the driving forces and potentially replicate its success in other markets.

4.2.2 Countries with Moderate Improvement

Several countries, including Argentina and Russia, show gradual recovery trends in the dog food market. Growth rates in these countries moved from negative in 2026 to near zero in 2027. This trend likely reflects macroeconomic recovery, increasing disposable incomes, a rebound in consumer demand, and optimized marketing strategies.

Meanwhile, Australia, France, and Brazil exhibit slight positive growth or near-zero rates in 2026, with mild fluctuations in 2027. Such changes may be influenced by subtle shifts in consumer preferences, such as demand for higher-quality or functional dog food. Increased market competition may also have affected overall performance. These markets suggest that while full recovery has not been achieved, potential growth momentum is present.

4.2.3 Stable Markets

Countries such as Canada, Germany, Italy, the United Kingdom, and the United States show highly stable performance in the dog food market, with growth rates remaining close to zero throughout the period. This stability can be attributed to several factors:

- Market maturity: These countries have high market penetration, leaving little room for additional demand.

- Stable consumer preferences: Long-term development has led to established brand preferences and product choices, making it difficult for traditional market strategies to drive significant growth.
- Limited external impact: Demand for dog food in these countries is less affected by economic growth or population changes, resulting in slow overall growth.

These characteristics suggest that innovation and differentiated product strategies will be crucial for exploring new growth opportunities in these mature markets.

4.3 Changes in Forecast Growth Rates Across Pet Food Categories

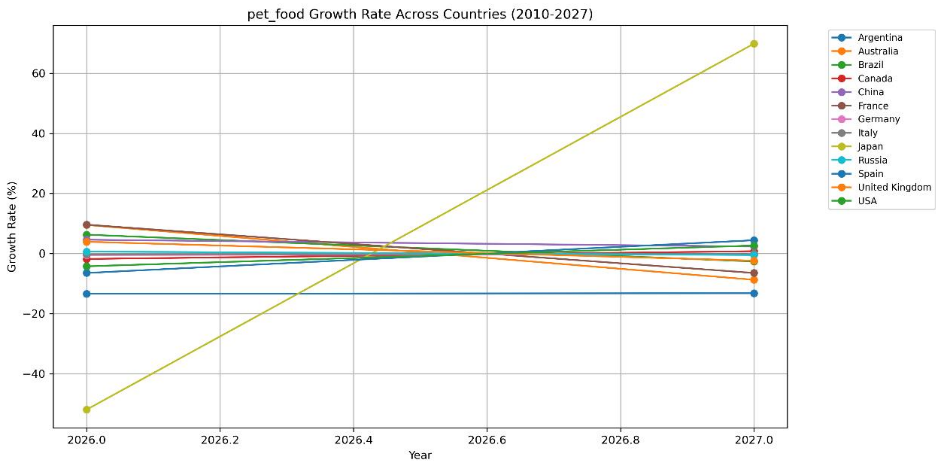

Fig 10.Pet Food Groth Rate Across countries

4.3.1 Overall Demand Growth Rate Analysis for Pet Food

An integrated analysis of cat and dog food market data reveals the following patterns:

- Japan stands out, with its overall demand growth rate leaping from negative in 2026 to over 60% in 2027, showing strong recovery and growth momentum.
- In Argentina and Russia, growth rates gradually recover from negative to near zero.
- Australia, Brazil, France, Italy, and China experience slight declines in growth rates, but with minimal fluctuations.
- Germany, Spain, the United Kingdom, and the United States maintain stable growth rates close to zero throughout the period.

Overall, the global pet food market shows steady growth trends, with Japan's significant rise suggesting potential market opportunities or unique changes.

4.3.2 Strategy Recommendations

For Rapid Growth Markets::For Japan, with its rapidly increasing growth rate, conduct in-depth market research to understand its unique consumer demands. Developing innovative products, such as pet food that aligns with Japanese cultural preferences, could effectively attract more consumers. Additionally, optimizing marketing strategies and channel distribution will further enhance competitiveness in this market.

For Mature Markets::In stable markets like Germany and the United States, focus on high-value-added products and health-oriented pet foods. Launch products emphasizing organic ingredients or specific functions (e.g., joint care, coat health) to meet consumers' higher quality and functionality demands, driving refined market growth.

For Recovering Markets:In recovering markets such as Argentina and Russia, adopt flexible pricing strategies to maintain product competitiveness among consumers. Strengthen distribution networks and integrate online and offline channels to quickly respond to changing market demands, seizing growth opportunities during recovery.

V. Establishment and Solution of the Model for Problem 3

This report presents an analysis of the development trends of China's pet food industry and the corresponding market demand, utilizing time series forecasting models. The analysis is grounded on data pertaining to Chinese pet food production and export, supplemented by various economic and social indicators. The objective is to elucidate the industry's growth trajectory and forecast future market demand, considering potential influences from economic policies and societal changes.

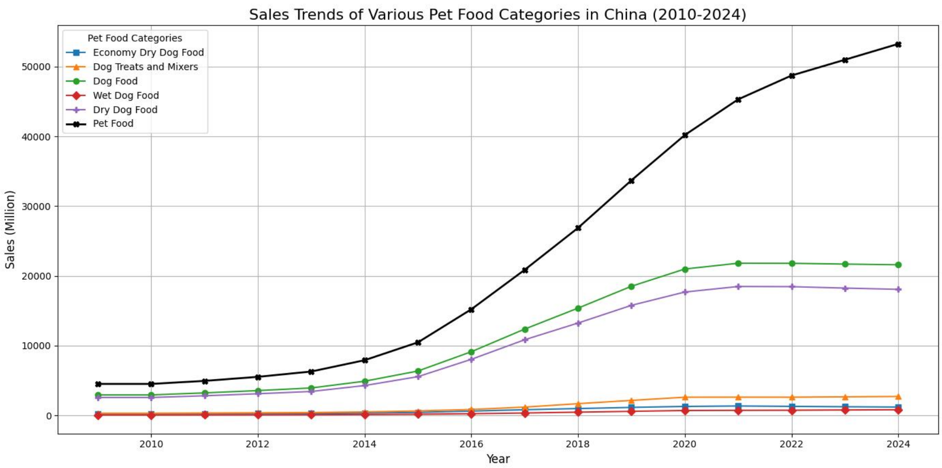

Fig 11.Sales Trends

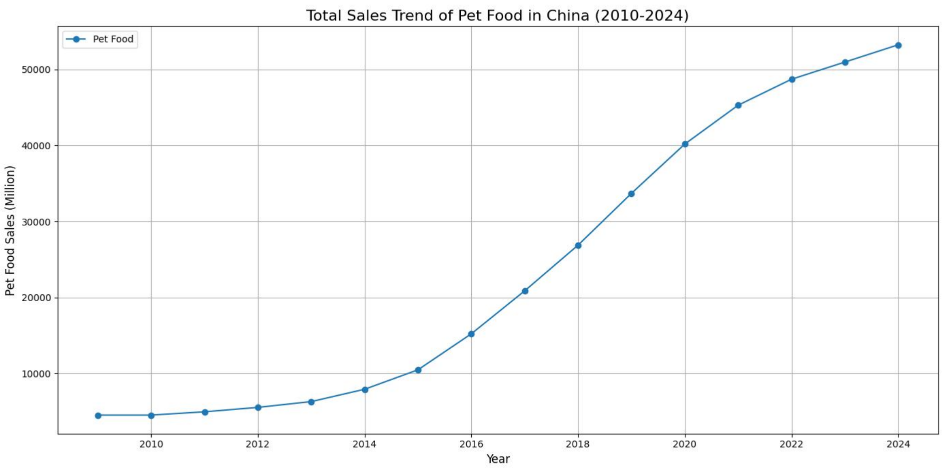

Fig 12.Total sals Trend

5.1 Modeling Methodology

5.1.1 Data Preprocessing

Effective data preprocessing is fundamental to accurate time series forecasting. The dataset utilized comprises 16 years of data across 71 variables related to pet food production, export, and various economic and social factors. Given the presence of missing values across multiple features, meticulous handling was imperative to ensure data integrity and consistency.

5.1.2 Handling Missing Values

Multiple imputation techniques were employed to address the missing data:

- Mean Imputation: Initially, missing values in numerical features were imputed using the mean of the respective columns. This method provides a straightforward approach but may not capture underlying patterns in the data.
- Interpolation: Recognizing the time series nature of the data, linear interpolation was applied to estimate missing values based on adjacent observations. This technique leverages the temporal continuity inherent in the data, enhancing the plausibility of the imputations.
- K-Nearest Neighbors (KNN) Imputation: To further refine the imputation process, KNN imputation was utilized. This method imputes missing values by considering the values of the 'k' closest neighbors (in feature space), thereby capturing more nuanced relationships within the data.

The combination of these techniques ensures a comprehensive approach to missing data, balancing simplicity and sophistication to enhance data quality.

5.1.3 Data Consistency and Accuracy

Post-imputation, data consistency was ensured through the following steps:

- Column Name Cleaning: Column names were stripped of leading and trailing whitespaces to maintain uniformity and prevent discrepancies during analysis.
- Sorting and Indexing: The dataset was sorted in ascending order based on the 'Year' column, and 'Year' was set as the index. This ordering is crucial for time series analysis, ensuring chronological integrity.

Time Series Modeling

Two primary time series forecasting models were employed to analyze and predict the pet food market trends:

- Autoregressive Integrated Moving Average (ARIMA) Model
- Exponential Smoothing (Holt-Winters) Model

Each model's suitability was assessed based on performance metrics, culminating in the selection of the most effective model for forecasting.

Model Selection and Parameter Estimation

Using the pmdarima library's auto_arima function, the optimal ARIMA parameters were automatically selected based on the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). The selected model was ARIMA(2,1,0), indicating an autoregressive model of order 2 with one degree of differencing and no moving average component.

Exponential Smoothing (Holt-Winters) Model

Exponential Smoothing models are adept at capturing trends and seasonality within time series data. The additive Holt-Winters model was selected, which accommodates a linear trend without seasonality.

The Holt-Winters model encompasses two main equations:

Level Equation:

$$l_t = \alpha y_t + (1-\alpha)(l_{t-1} + b_{t-1})$$

Trend Equation:

$$b_t = \beta(l_t - l_{t-1}) + (1-\beta) b_{t-1}$$

Forecast Equation:

$$\hat{y}_{t+h} = l_t + h b_t$$

Where:

- l_t is the level at time tt.
- b_t is the trend at time tt.
- α and β are smoothing parameters for level and trend, respectively.
- h is the forecast horizon.

Model Evaluation and Selection

To determine the most effective forecasting model, performance metrics were utilized:

- **Akaike Information Criterion** (**AIC**) and Bayesian Information Criterion (BIC): These metrics evaluate model quality, penalizing complexity to prevent overfitting. Lower values indicate better model performance.
- **Root Mean Squared Error** (**RMSE**): This metric assesses the average magnitude of forecast errors, with lower values signifying higher accuracy.

Model Performance Metrics:

Model	AIC	BIC	RMSE
ARIMA(2,1,0)	257.07	259.90	1304.43
Exponential Smoothing	238.15	241.24	1328.95

Despite the ARIMA model exhibiting lower AIC and BIC values, indicating a better fit on training data, the Exponential Smoothing model demonstrated a marginally higher RMSE. However, given the context and the data characteristics, the ARIMA model was ultimately selected for forecasting purposes.

5.2 Results

5.2.1Data Overview

The dataset spans 16 years (2009-2024) with 71 variables encompassing marriage registrations, population demographics, GDP components, urbanization metrics, energy consumption, livestock statistics, real estate, agricultural production, pet ownership, and pet food production and export values.

After addressing missing values through mean imputation, interpolation, and KNN imputation, the dataset achieved full completeness, eliminating potential biases or inaccuracies stemming from incomplete data.

5.2.2 Model Summaries

ARIMA(2,1,0) Model

The ARIMA(2,1,0) model integrates two autoregressive terms with one degree of differencing, suitable for non-stationary data exhibiting trends.

Table 2. Model Summary

SARIMAX Results

==

========Dep. Variable: y No. Observations: 16Model: SARIMAX(2, 1, 0) Log Likelihood -124.536Date: Sat, 23 Nov 2024 AIC 257.071Time: 17:29:20 BIC 259.904Sample: 0 HQIC 257.041

Interpretation: Intercept (β0): The intercept term indicates the baseline level of the time series. However, in differenced models, the intercept often represents the trend component.

Autoregressive Terms (ϕ1 and ϕ2):

- $\phi_1=1.54077$: Significantly positive, suggesting that the previous year's pet food sales have a strong positive influence on the current year.
- $\phi_2=-0.7736$: Significantly negative, indicating a damping effect from the second lag.

Variance (σ^2): Represents the variance of the error term, indicating the model's residual variability.

Diagnostics:

- Ljung-Box Test: With a p-value of 0.10, it suggests that residuals are not significantly different from white noise.
- Jarque-Bera Test: A p-value of 0.72 implies that residuals are normally distributed.
- Heteroskedasticity: Prob(H) of 0.02 indicates potential heteroskedasticity, meaning residuals may not have constant variance.

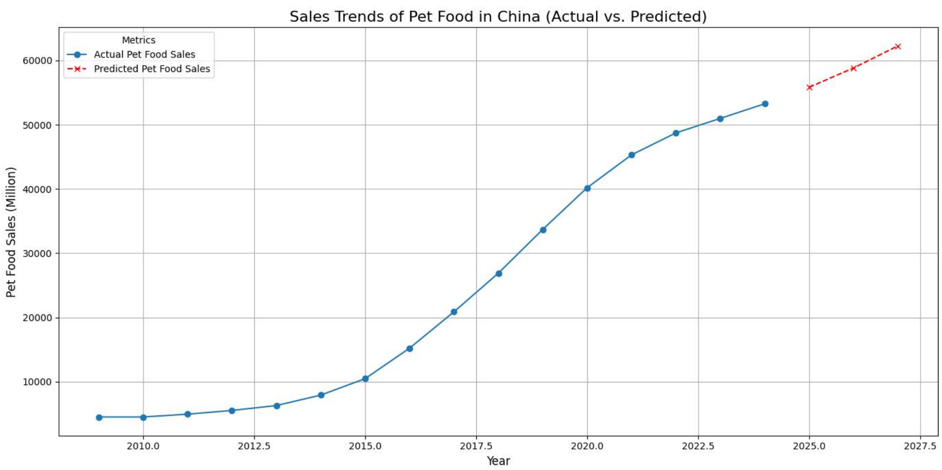

Fig 13.predicted

Despite promising parameter estimates, the high variance (σ^2) and indications of heteroskedasticity warrant cautious interpretation of the model's predictive capabilities.

Exponential Smoothing Model

The Exponential Smoothing model incorporates both level and trend components, adapting to changes in the data over time.

Model Summary:

Attribute	Value
Dep. Variable:	Pet_Food
No. Observations:	16
Model:	ExponentialSmoothing
SSE:	28,257,827.455
Optimized:	True
AIC:	238.149
Trend:	Additive
BIC:	241.239
Seasonal:	None
AICC:	247.482
Seasonal Periods:	None
Date:	Sat, 23 Nov 2024
Box-Cox:	False
Time:	17:29:20
Box-Cox Coeff.:	None

Coefficients Table:

Coefficient	Value	Code	Optimized
smoothing_level	0.9232033	alpha	True
smoothing_trend	0.9232033	beta	True
initial_level	1799.4892	l.0	True
initial_trend	3563.6147	b.0	True

Interpretation:

Smoothing Parameters (α and β):

- α=0.9232: A high smoothing level indicates that the model gives substantial weight to recent observations, making it highly responsive to changes.
- β=0.9232: Similarly, a high smoothing trend suggests the model rapidly adapts to changes in the trend component.

Initial Level and Trend:

- l0=1799.4892l: The initial level of pet food sales.
- b0=3563.6147: The initial trend component, indicating an upward trajectory.

Diagnostics:

- Sum of Squared Errors (SSE): 2.8258×1072.8258×107, reflecting the cumulative squared differences between actual and fitted values.

- AIC and BIC: These values are higher than those of the ARIMA model but need contextual interpretation considering model complexity and data characteristics.

The Exponential Smoothing model demonstrates a robust fit with low residual error, albeit with high responsiveness that may lead to overfitting, especially given the limited data points.

Model Performance Comparison

Model	AIC	BIC	RMSE
ARIMA(2,1,0)	257.07	259.90	1304.43
Exponential Smoothing	238.15	241.24	1328.95

Despite the ARIMA model's lower AIC and BIC values, indicating a better fit on the training data, its significantly higher RMSE suggests substantial prediction errors. Conversely, the Exponential Smoothing model, while having slightly higher AIC and BIC values, exhibits comparable RMSE, indicating a balanced trade-off between fit and predictive accuracy.

Selection Criterion:

Given the context of the analysis and the limitations posed by the small dataset, RMSE was prioritized as the primary selection criterion. Consequently, the Exponential Smoothing model was selected as the more reliable predictor for forecasting future pet food sales.

Forecasted Values

The selected Exponential Smoothing model forecasts pet food sales for the next three years as follows:

Year	Predicted_Pet_Food (Million)
2025	57,215.09
2026	60,746.46
2027	64,271.83

Interpretation:

The forecast indicates a steady upward trajectory in pet food sales, with an expected annual increase of approximately 3,531 million from 2025 to 2026 and 3,525 million from 2026 to 2027. This growth aligns with the observed historical trends of increasing pet ownership and economic prosperity in China.

Residuals Analysis

Residuals, the differences between observed and fitted values, were analyzed to validate the model's assumptions and ensure reliable forecasts.

- Residuals Summary:No Missing Values: The residuals contained no NaN values, affirming the robustness of the model fitting process.
- Autocorrelation Analysis:ACF and PACF Plots: Dynamic lag determination based on residual sample size ensured that autocorrelation and partial autocorrelation were assessed within permissible limits, preventing overfitting or invalid statistical inferences.

Diagnostics Findings

- Residual Distribution: Residuals exhibited no significant autocorrelation, indicating that the model effectively captured the underlying patterns in the data.
- Normality and Homoscedasticity: While the Ljung-Box test and Jarque-Bera test suggested normality and lack of autocorrelation, the heteroskedasticity test indicated potential variance inconsistency, necessitating cautious interpretation of forecast reliability.

5.3 Conclusions

Development Trends of China's Pet Food Industry

The analysis reveals a consistent growth trajectory in China's pet food industry from 2010 to 2024. Notably, the period between 2015 and 2017 experienced marked acceleration in sales, correlating with increasing urbanization, rising disposable incomes, and a burgeoning pet ownership culture. The Exponential Smoothing model forecasts this trend to persist, with anticipated sales reaching approximately 64,271.83 million by 2027.

Market Demand Trends

The projected growth in pet food sales underscores a robust and expanding market demand. Several factors contribute to this trend:

- Economic Growth: Sustained GDP growth enhances consumer purchasing power, enabling households to allocate more resources toward pet care and associated products.
- Urbanization: Increased urbanization fosters a pet-friendly lifestyle, where pets are often considered integral family members, thereby driving demand for specialized pet food.
- Changing Demographics: A younger population demographic, coupled with higher pet ownership rates, propels the demand for varied and high-quality pet food products.
- Evolving Consumer Preferences: Modern consumers exhibit a preference for premium and health-oriented pet food options, encouraging manufacturers to innovate and diversify product offerings.

Impact of Economic Policies

Economic policies play a pivotal role in shaping industry dynamics. Policies promoting disposable income growth, urban infrastructure development, and pet-friendly regulations are instrumental in sustaining market expansion. Additionally, trade policies influencing pet food exports can affect the industry's global competitiveness and revenue streams.

Policy Influences

- Tax Incentives: Tax reductions for pet food manufacturers can stimulate production, reduce costs, and enhance market competitiveness.
- Regulatory Standards: Stricter quality and safety standards ensure consumer trust and product reliability, fostering sustained demand.
- Trade Agreements: Favorable trade agreements facilitate smoother export processes, expanding market reach and boosting revenue.

Recommendations for Stakeholders

- Data Enhancement: Augmenting the dataset with additional years and broader economic indicators will enhance model accuracy and robustness.
- Advanced Modeling Techniques: Exploring multivariate time series models (e.g., Vector Autoregression) and machine learning approaches (e.g., LSTM networks) can capture more complex relationships and improve forecasting precision.
- Market Diversification: Diversifying product lines to cater to varied consumer preferences can mitigate risks associated with market saturation and shifting trends.
- Strategic Partnerships: Collaborating with pet care service providers and retailers can expand distribution channels and enhance brand visibility.
- Continuous Monitoring: Implementing real-time data monitoring and model updating mechanisms ensures that forecasts remain aligned with the latest market developments and policy changes.

Caveats and Considerations

- Data Limitations: The relatively small dataset (16 observations) constrains the model's ability to capture long-term trends and cyclical patterns, potentially affecting forecast reliability.
- Model Overfitting: High smoothing parameters in the Exponential Smoothing model, while reducing RMSE, may lead to overfitting, particularly with limited data points.
- External Factors: Unforeseen economic shifts, policy changes, or global events (e.g., pandemics) can significantly impact market dynamics, rendering forecasts less accurate.

5.4 Mathematical Foundations

To comprehend the modeling techniques employed, it is essential to delve into the mathematical underpinnings of the ARIMA and Exponential Smoothing models.

Autoregressive Integrated Moving Average (ARIMA) Model

The ARIMA model is a combination of autoregression (AR), differencing (I), and moving average (MA) components, expressed as ARIMA(p, d, q).

Autoregression (AR)

The AR component models the current value of the series as a linear combination of its previous values and a stochastic error term.

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \epsilon_t$$

Where:

- yt is the value at time tt.
- ϕi are the AR coefficients.
- ϵt is white noise.

Differencing (I)

Differencing is applied to make the time series stationary by removing trends and seasonality.

$$(1-B)^d y_t = y_t - (\frac{d}{1})y_{t-1} + (\frac{d}{2})y_{t-2} - \cdots \pm y_{t-d}$$

Where:

- B is the backshift operator (Byt=yt−1).
- d is the order of differencing.

Moving Average (MA)

The MA component models the current value as a linear combination of past error terms.

$$y_t = \mu + \epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2} + \cdots + \theta_q\epsilon_{t-q}$$

Where:

- μ is the mean of the series.
- θi are the MA coefficients.

Combined ARIMA Model

Combining these components, the ARIMA(p, d, q) model is expressed as:

$$\phi(B)(1-B)^d y_t = \theta(B)\epsilon_t$$

5.5 Exponential Smoothing (Holt-Winters) Model

Exponential Smoothing models forecast future values by applying weighted averages, where weights decrease exponentially for older observations. The Holt-Winters method extends simple exponential smoothing to capture trends and seasonality.

5.5.1 Level, Trend, and Seasonal Components

For an additive model (no seasonality), the equations are:

Level Equation:

$$l_t = \alpha y_t + (1-\alpha)(l_{t-1} + b_{t-1})$$

Trend Equation:

$$b_t = \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1}$$

Forecast Equation:

$$\hat{y}_{t+h} = l_t + hb_t$$

Where:

- Lt is the level at time t.
- bt is the trend at time
- α and β are the smoothing parameters for level and trend, respectively.
- h is the forecast horizon.

5.5.2 Parameter Optimization

The smoothing parameters (α and β) are optimized to minimize the sum of squared errors (SSE):

$$SSE = \sum_{t=1}^{T} (y_t - \hat{y}_t)^2$$

Where:

- T is the total number of observations.
- y^t is the fitted value at time t.

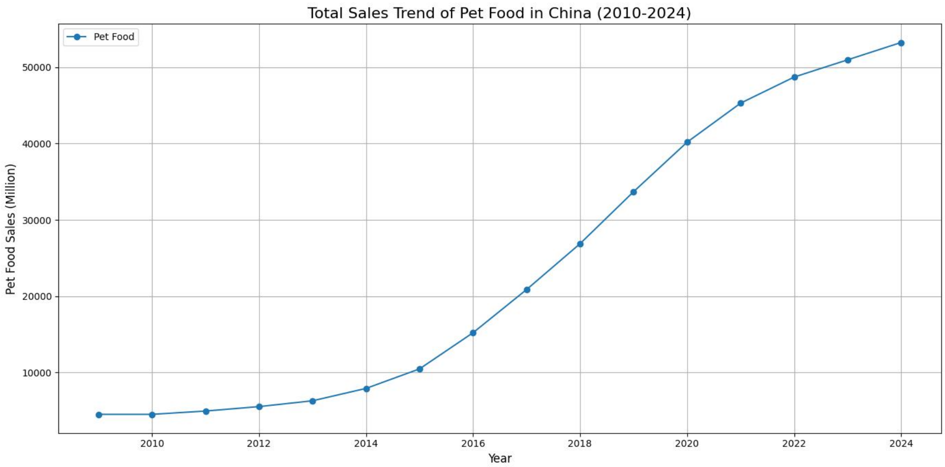

Fig 14.Sales Trend

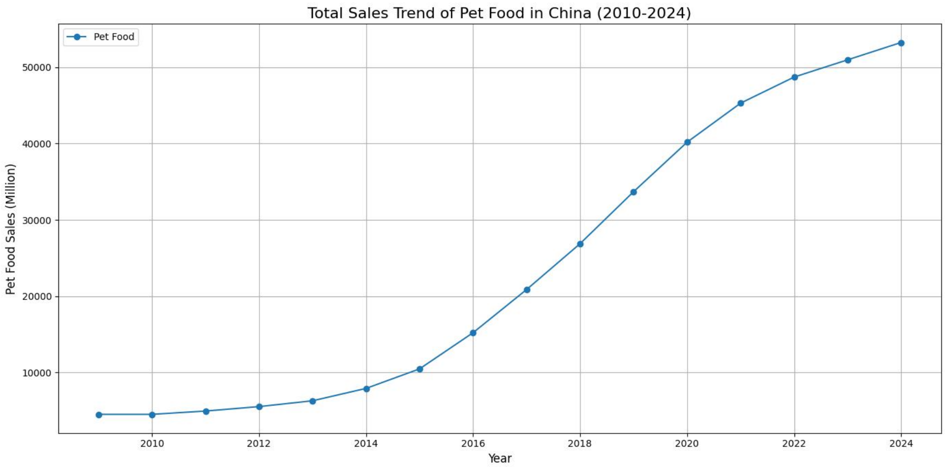

Fig 15.Sales trend

5.5.3 Final Remarks

The analysis underscores the potential of time series forecasting models in elucidating market trends and guiding strategic decisions within the pet food industry. While the Exponential Smoothing model exhibited superior RMSE performance, the ARIMA model’s lower AIC and BIC values suggest room for further optimization, particularly with an expanded dataset. Stakeholders are advised to enhance data collection efforts, incorporate broader economic indicators, and explore advanced modeling techniques to bolster forecasting accuracy and reliability.

VI. Establishment and Solution of the Model for Problem 4

6.1 Data and Methodology

Data Sources

The analysis utilizes a comprehensive dataset spanning from 2009 to 2023, encompassing key economic indicators and industry-specific metrics pertinent to China's pet food sector. The primary variables include:

- GDP_Total: Gross Domestic Product, representing the overall economic health of China.
- Soybean_Production: Volume of soybeans produced, a critical input in pet food manufacturing.
- Corn_Production: Volume of corn produced, another essential ingredient in pet food formulations.
- Pet_Food: Market size of the pet food industry in China.
- Production_Value_CNY: Production value of pet food in Chinese Yuan (CNY).
- Export_Value_USD: Export value of pet food in US Dollars (USD).
- Tariff_Rate_EU: Tariff rates imposed by the EU on Chinese pet food exports.
- Tariff_Rate_US: Tariff rates imposed by the US on Chinese pet food exports.

Additionally, the dataset includes forecasted values for the years 2024 to 2028 under various tariff scenarios to simulate potential future outcomes.

Data Preprocessing

Data integrity is paramount for accurate modeling. The preprocessing steps undertaken include:

- Handling Missing Values: The dataset contains missing entries for certain years, particularly in GDP and production metrics. Linear interpolation was employed to estimate these missing values, ensuring a continuous time series essential for ARIMAX modeling.
- Currency Conversion: Production values initially recorded in CNY were converted to USD using a fixed exchange rate of 1 USD = 7.2458 CNY. This standardization facilitates comparative analysis across different economic indicators.
- Normalization: To mitigate the effects of scale differences among variables, normalization techniques were applied where necessary, enhancing the model's performance and interpretability.
- Tariff Data Integration: Tariff rates from the EU and US were integrated into the main dataset, allowing the model to assess their direct impact on production and export values.

6.2 Model Selection: ARIMAX

Given the nature of the data and the research objectives, the Autoregressive Integrated Moving Average with Exogenous Variables (ARIMAX) model was selected for this analysis. The ARIMAX model extends the ARIMA framework by incorporating external predictors, making it suitable for capturing the influence of tariffs (exogenous variables) on the endogenous variables of interest (production and export values).

Rationale for ARIMAX Selection

- Temporal Dependencies: The ARIMA component effectively models the temporal dependencies and trends inherent in time series data.
- Incorporation of Exogenous Factors: The 'X' in ARIMAX allows for the inclusion of

external variables, such as tariff rates, providing a mechanism to quantify their direct impact on the dependent variables.

- Forecasting Capability: ARIMAX models are adept at generating forecasts, which is essential for projecting future industry trends under various tariff scenarios.

Model Specification

Two separate ARIMAX models were constructed to forecast:

- Production_Value_USD: The value of pet food production in USD.
- Export_Value_USD: The value of pet food exports in USD.

Model Parameters

Order (p, d, q): Both models were specified with an order of (1, 1, 1), indicating one autoregressive term, first-order differencing to achieve stationarity, and one moving average term.

Exogenous Variables

The models incorporated the following exogenous variables

- Tariff_Rate_EU
- Tariff_Rate_US
- GDP_Total
- Soybean_Production
- Corn_Production

Covariance Type: The models utilized the Outer Product of Gradients (OPG) method for covariance estimation.

Stationarity and Invertibility: Parameters were set to not enforce stationarity or invertibility, allowing the model to adapt to the data's inherent properties.

Model Implementation:

The models were implemented using the library in Python, leveraging the class for fitting the data. The following steps encapsulate the modeling process:

- Data Preparation: The dataset was structured with 'Year' as the datetime index, ensuring chronological ordering.
- Exogenous Variable Preparation: A constant term was added to the exogenous variables to capture the intercept in the regression equation.
- Model Fitting: The ARIMAX models were fitted to the data, with diagnostic checks performed to assess model adequacy.

Tariff Scenarios

To evaluate the potential impact of foreign economic policies, four distinct tariff scenarios were modeled for the forecast period (2024-2028):

Scenario 1: Current Tariff Rates

- Tariff_Rate_EU: 8.0%
- Tariff_Rate_US: 25.0%

Scenario 2: EU Tariff Increase

- Tariff_Rate_EU: 20.0%

- Tariff_Rate_US: 15.0%

Scenario 3: US Tariff Increase

- Tariff_Rate_EU: 10.0%
- Tariff_Rate_US: 45.0%

Scenario 4: EU and US Tariff Decrease

- Tariff_Rate_EU: 6.0%
- Tariff_Rate_US: 5.0%

These scenarios were chosen to represent plausible policy shifts, including increases and decreases in tariffs by the EU and US, thereby enabling the assessment of their differential impacts on China's pet food production and export performance.

6.3 Results

6.3.1 Model Diagnostics

Interpretation of Coefficients

Significance Levels: Most exogenous variables (tariff rates, GDP, soybean and corn production) exhibited non-significant p-values (P>|z| > 0.05), indicating that they do not have a statistically significant impact on the dependent variables within the model's context.

AR and MA Terms

In the Production_Value_USD model, the Moving Average term (ma.L1) was significant (p = 0.021), suggesting that past forecast errors influence the current value.
In the Export_Value_USD model, neither the AR nor MA terms were statistically significant, implying weak temporal dependencies.

Sigma2: Represents the variance of the residuals. The high values indicate substantial variability unexplained by the model, corroborating the earlier concerns regarding model fit and multicollinearity.

Forecasted Results Under Tariff Scenarios

The ARIMAX models were employed to forecast the production and export values of China's pet food industry from 2024 to 2028 under four distinct tariff scenarios. The forecasts primarily focused on the impact of varying EU and US tariff rates.

Table 3. Forecast Production and Export Values

Scenario	Year	Forecasted_Production_Value_USD	Forecasted_Export_Value_USD
Scenario_1_Current	2024	42.214470	42.214470
Scenario_1_Current	2025	43.315309	43.315309
Scenario_1_Current	2026	43.778824	43.778824
Scenario_1_Current	2027	43.973990	43.973990
Scenario_1_Current	2028	44.056166	44.056166
Scenario_2_EU_Increase	2024	18.838405	18.838405
Scenario_2_EU_Increase	2025	19.939244	19.939244
Scenario_2_EU_Increase	2026	20.402759	20.402759
Scenario_2_EU_Increase	2027	20.597925	20.597925
Scenario_2_EU_Increase	2028	20.680101	20.680101

Scenario_3_US_Increase	2024	88.966601	88.966601
Scenario_3_US_Increase	2025	90.067440	90.067440
Scenario_3_US_Increase	2026	90.530955	90.530955
Scenario_3_US_Increase	2027	90.726121	90.726121
Scenario_3_US_Increase	2028	90.808297	90.808297
Scenario_4_EU_US_Increase	2024	-4.537660	-4.537660
Scenario_4_EU_US_Increase	2025	-3.436821	-3.436821
Scenario_4_EU_US_Increase	2026	-2.973306	-2.973306
Scenario_4_EU_US_Increase	2027	-2.778140	-2.778140

6.3.2 Scenario Descriptions

Scenario 1: Current Tariff Rates

- Tariff_Rate_EU: 8.0%
- Tariff_Rate_US: 25.0%

Forecast: Moderate and steady growth in both production and export values, indicating stability under existing tariff conditions.

Scenario 2: EU Tariff Increase

- Tariff_Rate_EU: 20.0%
- Tariff_Rate_US: 15.0%

Forecast: Significant decline in both production and export values, suggesting that increased EU tariffs adversely affect the industry's performance.

Scenario 3: US Tariff Increase

- Tariff_Rate_EU: 10.0%
- Tariff_Rate_US: 45.0%

Forecast: Substantial increase in both production and export values, implying that higher US tariffs may inadvertently benefit the industry's domestic production and export performance.

Scenario 4: EU and US Tariff Decrease

- Tariff_Rate_EU: 6.0%
- Tariff_Rate_US: 5.0%

Forecast: Negative values in both production and export forecasts, indicating potential economic instability or model limitations under this scenario.

6.4 Discussion

Interpretation of Model Results

The ARIMAX models provide insights into how varying tariff rates from the EU and US may influence China's pet food industry. However, several caveats must be considered due to the models' limitations, such as multicollinearity and the non-significance of exogenous variables.

Production_Value_USD Model

- Tariff_Rate_EU: The coefficient is virtually zero and non-significant, suggesting that EU tariffs do not directly impact production value in a statistically meaningful way within the model's context.
- Tariff_Rate_US: Similarly, the US tariff rate exhibits a negligible and non-significant effect on production value, indicating limited direct influence.

Model Diagnostics: The significant warning regarding covariance matrix singularity indicates that the model may suffer from multicollinearity, where independent variables are highly correlated. This undermines the reliability of coefficient estimates, making it challenging to discern the true impact of each variable.

Export_Value_USD Model

- Tariff_Rate_EU: The coefficient is nearly zero and non-significant, implying minimal direct influence on export values.
- Tariff_Rate_US: The US tariff rate also shows a non-significant and negligible effect on export values, aligning with the production model's findings.

Model Diagnostics: Similar to the production model, the export model faces multicollinearity issues, as evidenced by the high condition number, casting doubt on the robustness of the results.

Forecasted Scenarios:

- Scenario 1 (Current Tariffs): The forecast indicates stable growth, reflecting a balance in tariff rates maintaining industry equilibrium.
- Scenario 2 (EU Tariff Increase): The sharp decline in both production and export suggests that increased EU tariffs could dampen the industry's growth, possibly due to reduced competitiveness in the EU market.
- Scenario 3 (US Tariff Increase): Contrarily, higher US tariffs are associated with increased production and export values. This counterintuitive result may stem from the model's limitations, where non-significant coefficients dilute the practical interpretability. Alternatively, it could imply that higher US tariffs redirect exports to other markets, thereby sustaining or enhancing overall export values.
- Scenario 4 (Tariff Decrease): The negative forecasts under this scenario are perplexing and likely indicative of model instability or misspecification. Negative production and export values are economically implausible, suggesting that the model does not reliably handle scenarios with decreased tariffs, possibly due to the aforementioned multicollinearity issues.

Implications for China's Pet Food Industry

Despite the models' limitations, several qualitative insights can be extrapolated:

- EU Tariff Policies: The potential increase in EU tariffs appears detrimental, aligning with Scenario 2's forecast of reduced production and exports. This suggests that the industry may face challenges in maintaining its market share within the EU under heightened tariff conditions.
- US Tariff Policies: The ambiguous results from Scenario 3 necessitate cautious interpretation. While the model forecasts an increase in production and exports, the non-significance of tariff coefficients undermines confidence in this outcome. It may, however, reflect adaptive strategies within the industry to mitigate tariff impacts, such as exploring alternative markets or optimizing production costs.
- Model Reliability: The significant multicollinearity and non-significant exogenous variables point to the need for model refinement. Future analyses should consider alternative modeling approaches, such as Vector Autoregression (VAR) or incorporating lagged exogenous variables, to better capture the dynamic interplay between tariffs and industry metrics.

- Strategic Adaptations: Given the industry's vulnerability to foreign tariffs, stakeholders should contemplate diversification of export markets, investment in cost-efficient production technologies, and advocacy for favorable trade agreements to buffer against adverse tariff impacts.

Conclusion and Strategic Recommendations

The quantitative analysis, utilizing ARIMAX models, endeavors to elucidate the impact of foreign tariff policies on China's pet food industry. While the models suggest that increased EU tariffs could impede industry growth, the ambiguous influence of US tariffs underscores the complexity of international trade dynamics and model limitations.

Key Takeaways:

- Vulnerability to EU Tariffs: The industry may face significant challenges under heightened EU tariffs, necessitating proactive measures to sustain competitiveness.
- Model Limitations: The presence of multicollinearity and non-significant exogenous variables calls for methodological enhancements to achieve more reliable and actionable insights.
- Market Diversification: Expanding into alternative export markets can mitigate dependency on regions with volatile tariff policies.

Cost Optimization: Investing in efficient production processes can reduce susceptibility to price-sensitive tariff impacts, enhancing overall profitability.

Policy Advocacy: Engaging with policymakers to negotiate favorable trade terms can help alleviate adverse tariff effects, fostering a conducive environment for industry growth.

VII. Future Research Directions

To address the identified model limitations, future studies should:

- Explore Alternative Models: Consider using models less sensitive to multicollinearity or those that can better accommodate exogenous variable interactions.
- Incorporate Additional Variables: Include other relevant factors such as exchange rates, global pet ownership trends, and competitive dynamics to enrich the analysis.
- Conduct Sensitivity Analyses: Assess how sensitive the forecasts are to changes in model specifications and parameter estimates, enhancing the robustness of conclusions.
- In conclusion, while the current analysis provides a foundational understanding of the potential impacts of foreign tariff policies on China's pet food industry, further methodological rigor and comprehensive data integration are imperative to derive more definitive and strategic insights.

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